

Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Chapter Quiz – Sequences & Series

Questions

Q1.

A sequence of numbers u_1, u_2, u_3, \dots satisfies

$$u_{n+1} = 2u_n - 6, \quad n \geq 1$$

Given that $u_1 = 2$

(a) find the value of u_3

(2)

(b) evaluate $\sum_{i=1}^4 u_i$

(3)

(Total for question = 5 marks)

Q2.

An arithmetic series has first term a and common difference d .

Given that the sum of the first 9 terms is 54

(a) show that

$$a + 4d = 6 \tag{2}$$

Given also that the 8th term is half the 7th term,

(b) find the values of a and d .

(4)

(Total for question = 6 marks)

Q3.

A business is expected to have a yearly profit of £275 000 for the year 2016. The profit is expected to increase by 10% per year, so that the expected yearly profits form a geometric sequence with common ratio 1.1

(a) Show that the difference between the expected profit for the year 2020 and the expected profit for the year 2021 is £40 300 to the nearest hundred pounds.

(3)

(b) Find the first year for which the expected yearly profit is more than one million pounds.

(4)

(c) Find the total expected profits for the years 2016 to 2026 inclusive, giving your answer to the nearest hundred pounds.

(3)

(Total for question = 10 marks)

Q4.

The first term of a geometric series is 20 and the common ratio is 0.9

(a) Find the value of the fifth term.

(2)

(b) Find the sum of the first 8 terms, giving your answer to one decimal place.

(2)

Given that $S_{\infty} - S_N < 0.04$ where S_N is the sum of the first N terms of this series,

(c) show that $0.9^N < 0.0002$

(4)

(d) Hence find the smallest possible value of N .

(2)

(Total for question = 10 marks)